HEATING LONG OBJECTS IN THROUGH CONTACT INSTALLATIONS

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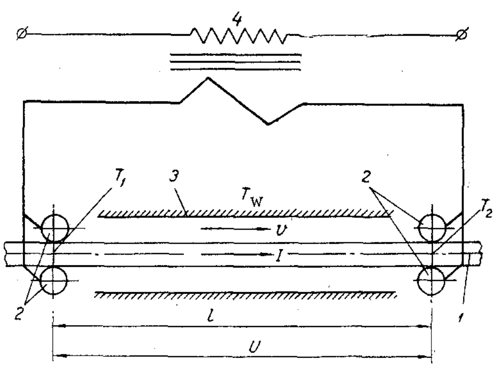
A method is suggested for estimating the electrical and thermal parameters in electrocontact installations. Calculated and experimental values for heating a steel strip are compared.

The method of heating metal parts by passing an electric current through them is in widespread industrial use [1]. Besides equipment where the object being heated is clamped in fixed contacts, one frequently meets installations in which a long object is continuously and uniformly moved over current-carrying rolls (or liquid contacts) and is brought to the required temperature over the section between them (see diagram).



Schematic of a through contact system.

This type of apparatus is used for wire or steel strip, but can also be used for heat-treating pipe produced in continuous rolling mills.

Existing methods of calculating electrical and thermal parameters are based on very conditional assumptions which either apply to specialized heating requirements [2, 3] or make use of elementary engineering conclusions [4] that do not meet the practical requirements of precision heating and are inadequate for large-scale machines dealing with a wide variety of materials.

We suggest a method, based on solving the heat balance equation, which will give more accurate values for the basic thermal parameters.

The diagram shows schematically an object 1, moving through current-carrying rolls 2 at a velocity $v = l/\tau_k$ in a heat-insulated muffle furnace 3 with wall temperature T_W. Power U is delivered from a transformer 4 to the rolls. A current i passes through the section of the object being heated l. This raises the temperature from T_1 to T_2.

We ignore heat losses in the contacts and consider the amount of heat in the section dl per unit time for an object with a constant cross-sectional area S and perimeter p, made of material whose density is γ, resistivity $\rho = a + bt$ [5, 6], average heat capacity c and apparent radiation coefficient σ_B [5].

The heat balance equation in this instance is

$$I^2 \rho \frac{dl}{S} = c\gamma S v dT + \sigma_B (T^4 - T_W^4)\, p dl. \qquad (1)$$

After the substitution

$$a + bt = (a - 273b) + bT$$

and a series of transformations

$$dK = \frac{\sigma_B T_W^3}{c\gamma R}\, d\tau, \quad \Omega = \frac{I^2 b}{p^2 \sigma_B T_W^3 R}, \quad R = \frac{S}{p},$$

$$\Theta_\Delta = \frac{1}{T_W}\left(\frac{a}{b} - 273\right), \quad \theta = \frac{T}{T_W}$$

we obtain

$$dK = \frac{d\theta}{1 + \Omega\Theta_\Delta + \Omega\theta - \theta^4}. \qquad (2)$$

To find the quadrature of (2), the polynomial in the denominator is decomposed into two quadratic trinomials by solving a cubic equation for the auxiliary parameter α [7]:

$$\alpha_0 = \left(\frac{\Omega^2}{16} + \sqrt{\frac{\Omega^4}{16^2} + \frac{(1+\Omega\Theta_\Delta)^3}{27}}\right)^{1/3} + \left(\frac{\Omega^2}{16} - \sqrt{\frac{\Omega^4}{16^2} + \frac{(1+\Omega\Theta_\Delta)^3}{27}}\right)^{1/3}. \qquad (3)$$

Then, for a temperature increase from T_1 to T_2 in time $\tau_K = vl$ expression (2) in integral from is written

$$K\Big|_0^{\tau_K} = -\int_{\theta_1}^{\theta_2} \{[\theta^2 - \sqrt{2\alpha_0}\,\theta + (\alpha_0 - \Omega/2\sqrt{2\alpha_0})] \times [\theta^2 + \sqrt{2\alpha_0}\,\theta + (\alpha_0 + \Omega/2\sqrt{2\alpha_0})]\}^{-1} d\theta. \qquad (4)$$

One of the solutions of integral (4) which is correct for the majority of metals and which is satisfied under the condition

$$2\alpha_0 - 2\Omega/\sqrt{2\alpha_0} < 0; \quad 2\alpha_0 + 2\Omega/\sqrt{2\alpha_0} > 0, \qquad (5)$$

is written in the following form:

$$K\Big|_0^{\tau_K} = \frac{1}{g^4 + 2h^2}\left(\frac{g}{2} \ln\left[\frac{\theta^2 - g\theta + g^2/2 - h}{\theta^2 + g\theta + g^2/2 + h}\right] - \right. \qquad (6)$$

$$-\frac{g^2-2h}{\sqrt{g^2+4h}}\times\operatorname{arctg}\frac{2\theta+g}{\sqrt{g^2+4h}}-$$

$$-\frac{g^2+2h}{2\sqrt{4h-g^2}}\ln\left[\frac{2\theta-g-\sqrt{4h-g^2}}{2\theta-g+\sqrt{4h-g^2}}\right]\Bigg)\Bigg|_{\theta_1}^{\theta_2}, \quad (6)\ \text{(contd)}$$

where $g=\sqrt{2\alpha_0}$; $h=\Omega/2\sqrt{2\alpha_0}$.

The last expression (6)

$$K=f_1(\theta,\ \Omega,\ \Theta_\Delta)$$

makes it possible to establish a connection between the heating time τ_K or the rate of displacement v, for a given distance l between rolls, and the heating temperature, the current I, and the electrical properties of the material.

Electrical resistance of the heated section is

$$r=\rho_{av}l/S,$$

where ρ_{av} is the mean resistivity over section l, which is estimated as

$$\rho_{av}=\int_0^l\frac{\rho\,dl}{l}=\int_0^{\tau_K}\frac{\rho\,d\tau}{\tau_K}. \quad (7)$$

From expression (2) we obtain

$$\frac{d\tau}{\tau_K}=\frac{1}{K|_{\tau=\tau_K}}\ \frac{d\theta}{1+\Omega\Theta_\Delta+\Omega\theta-\theta^4},$$

which, together with the known approximation, $\rho = a + bt = bT_W(\Theta_\Delta + \theta)$, after substitution in (7), gives

$$K|_{\tau=\tau_K}H=\int_{\theta_1}^{\theta_2}(\Theta_\Delta+\theta)\ \frac{d\theta}{1+\Omega\Theta_\Delta+\Omega\theta-\theta^4}, \quad (8)$$

where $H = \rho_{av}/bT_W$.

In the same way as we calculated the integral of expression (2), given conditions (5), we get for expression (8)

$$K|_{\tau=\tau_K}H=\frac{1}{g^2+2h^2}\left(\frac{g\Theta_\Delta-h}{2}\ln\left[\frac{\theta^2-g\theta+g^2/2-h}{\theta^2+g\theta+g^2/2+h}\right]+\right.$$

$$+\frac{g(g^2+h)-\Theta_\Delta(g^2-2h)}{\sqrt{g^2+4h}}\operatorname{arctg}\frac{2\theta+g}{\sqrt{g^2+4h}}- \quad (9)$$

$$\left.-\frac{g(g^2-h)+\Theta_\Delta(g^2+2h)}{2\sqrt{4h-g^2}}\ln\left[\frac{2\theta-g-\sqrt{4h-g^2}}{2\theta-g+\sqrt{4h-g^2}}\right]\right)\Bigg|_{\theta_1}^{\theta_2}.$$

The expression $KH=f_2(\theta,\ \Omega,\ \Theta_\Delta)$ enables us to establish the dependence of the mean resistivity on the quantities connected by relation (6).

Since expressions (6) and (9) are given in the form of dimensionless quantities, they can be used, after conversion to nomogram form, for rapid calculation of the parameters under practical operational conditions.

To estimate how closely the electrical and thermal parameters found by using the method outlined approximate to the actual values, we shall check the case described in [3] for steel strip with a carbon content of 0.11%.

Initial data. Dimensions of strip 80 × 8 mm; distance between rolls l = 1 m; end temperature of strip T_2 = 1073° K; rate of displacement v = 0.037 m/sec, which for l = 1 m corresponds to τ_K = 1/0.037 = 27 sec; voltage applied to rolls U = 12.5 V; voltage losses for cold rolls (T = 373° K) U_{cr} = 0.6 V; for hot rolls (T = 1073° K) U_{hr} = 2.8 V; current I = 12 000 A. We take the initial strip temperature T_1 = 293° K; temperature of surrounding medium T_W = 293° K and the following thermophysical properties of the steel [6]: C = 565 J/kg · deg; $\gamma = 75.5\times10^3$ N/m³; $\rho = 9\cdot10^{-3} + 0.12\cdot10^{-3}\,t$ = ohm · m; $\sigma_B = 5.53\cdot10^{-3}$ J/m² · sec × deg⁴, which is obtained by averaging and approximation in the given temperature range. The value of σ_B is given in [3].

We now make our calculations.

Current criterion:

$$\Omega=\frac{I^2b}{p^2\sigma_B T_W^3R}=\frac{12000^2\cdot0.12\cdot10^{-8}}{0.176^2\cdot5.53\cdot10^{-8}\cdot293^3\cdot3.63\cdot10^{-3}}=1110.$$

Criterion characterizing resistivity:

$$\Theta_\Delta=\frac{1}{T_W}\left(\frac{a}{b}-273\right)=\frac{1}{293}\left(\frac{9\cdot10^{-8}}{0.12\cdot10^{-8}}-273\right)=-0.675.$$

The auxiliary parameter

$$\alpha_0=58.8$$

is calculated from formula (3).

The quantities $\theta_1=1$, $\theta_2=3.66$, $g=10.85$, $h=51.1$ are determined from the formulas given in this paper.

Check for conditions (5):

$$2\alpha_0-2\Omega/\sqrt{2\alpha_0}=-86<0,\quad 2\alpha_0+2\Omega/\sqrt{2\alpha_0}=322>0.$$

The time criterion is calculated from formula (6):

$$\text{for } \theta=\theta_2\quad K_2=-1.12\cdot10^{-3},$$
$$\text{for } \theta=\theta_1\quad K_1=-3.42\cdot10^{-3}.$$

Then $K|_{\tau=\tau_K}=K_2-K_1=(-1.12+3.42)\cdot10^{-3}=2.3\cdot10^{-3}$.

Heating time

$$\tau_K=\frac{c\gamma R}{9.8\sigma_B T_W^3}(K_2-K_1)=$$

$$=\frac{565\cdot75.5\cdot10^3\cdot3.63\cdot10^{-3}\cdot2.3\cdot10^{-3}}{9.8\cdot5.53\cdot10^{-8}\cdot293^3}=26\text{ sec}, \quad (10)$$

τ_K obtained by calculation differs from the actual value by $\frac{27-26}{27}\times100=3.7\%$.

The product KH is determined from formula (9):

$$\text{for } \theta=\theta_2\quad (KH)_2=6.31\cdot10^{-3},$$

$$\text{for } \theta=\theta_1\quad (KH)_1=3.47\cdot10^{-3}.$$

Then

$$K|_{\tau=\tau_K} H = (KH)_2 - (KH)_1 = (6.31-3.47)\cdot 10^{-3} = 2.84\cdot 10^{-3},$$

$$H = \frac{KH}{K}\Big|_{\tau=\tau_K} = \frac{2.84\cdot 10^{-3}}{2.3\cdot 10^{-3}} = 1.235.$$

The mean resistivity

$$\rho_{av} = H b T_w = 1.235\cdot 0.12\cdot 10^{-8}\cdot 293 = 43.5\cdot 10^{-8}\ \text{ohm}\cdot\text{m}.$$

The resistance of heated section

$$r = \rho_{av} l/S = 43.5\cdot 10^{-8}\cdot 1/640\cdot 10^{-6} = 0.68\cdot 10^{-3}\ \text{ohm}.$$

The power delivered to the heated section l is calculated from the formula

$$W = \Omega H Q_{in}, \qquad (11)$$

where $Q_{in} = \sigma_B T_w^4 pl = 5.53\cdot 10^{-8}\cdot 293^4\cdot 0.176\cdot 1 = 72.5$ W, $W = 1110\cdot 1.235\cdot 72.5 = 100$ kW.

The voltage applied to the heating section is

$$U_l = \sqrt{Wr} = \sqrt{100\cdot 10^3\cdot 0.68\cdot 10^{-3}} = 8.25\ \text{V}.$$

The total calculated voltage applied to the rolls is

$$U = U_l + U_{cr} + U_{hr} = 8.25+0.6+2.8 = 11.65\ \text{V},$$

which varies from the actual value (allowing for errors in calculations and measurements) by $(11.65-12.5)/12.5\cdot 100 = 6.8\%$.

The thermal efficiency of the process in the heating section is

$$\eta = \Delta\theta/KH\Omega, \qquad (12)$$

where

$$\Delta\theta = (T_2 - T_1)/T_w = (1073-293)/293 = 2.66,$$

$$\eta = \frac{2.66}{2.3\cdot 10^{-3}\cdot 1110\cdot 1.235} = 0.84.$$

The small calculation errors make it possible to recommend formulas (6), (9), (10)–(12) for practical calculations of the electrical and thermal parameters with this type of heating. The method outlined can be used for any type of material.

All the parameters characterizing the heating process under examination are given as criterial relations. This makes it possible to construct nomograms from relations (6) and (9). These nomograms simplify any necessary calculation of the parameters for a particular heating process.

NOTATION

K – time criterion; Ω – current criterion; R – characteristic dimension; θ_Δ – criterion characterizing resistivity; H – resistance criterion.

REFERENCES

1. G. M. Tel'nov and E. I. Natanson, Electric Heating by the Resistance Method [in Russian], GONTI, 1951.
2. F. Knill, Elektrowärme, 3, 46–50, 1937.
3. B. Sochor and E. Kacki, Elektryka, no. 4, 1–14, 1958.
4. A. D. Svenchanskii, Electric Industrial Furnaces [in Russian], Gosenergoizdat, 1958.
5. V. P. Kozinets and G. N. Heifetz, collection: Electrothermy [in Russian], 34, 20–23, 1964.
6. Thermophysical Properties of Substances, handbook ed. N. B. Vagraftik [in Russian].
7. I. N. Bronshtein and K. A. Semendyaev, Handbook of Mathematics [in Russian], GITTL, 1958.

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